Brick and Block Masonry – Trends, Innovations and Challenges – Modena, da Porto & Valluzzi (Eds)
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The analysis of the main spire reversed arches of the Cathedral of Milan

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ABSTRACT: The recent restoration works on the main spire of the Cathedral of Milan involved the substitution of the old and corroded metallic tie rods and the neighbouring broken stone at the base, where the spire interacts with eight great reversed arches and buttresses made of Candoglia marble. The arches will be studied in relation to the observed damage and structural system configuration. The damage results from ageing of the structure, built in the 18th century, corrosion, vibrations, environmental conditions, such as strong wind, due to its height corresponding to 75–108 m above the ground. The stability will be evaluated by an analytical and simplified method based on the limit analysis principles through a static and kinematic approach. The combination of experimental and analytical approaches is used in understanding the damage phenomena and the structural behaviour of these elements to make an assessment of their safety more reliable.

1 INTRODUCTION

The Cathedral of Milan (Duomo di Milano), took five centuries to complete and is one of the largest churches in the world. The monumental structure is very complex, requiring much restoration work for different reasons, including exposure to the environmental ageing of the delicate Candoglia pink marble, the structural phenomena due to the construction phases, corrosion of iron members, vibration and recently, the great variation of the water table level during the 20th century.

Continuous restoration work to strengthen and replace the damaged parts is necessary, involving a numerous workforce to execute repair operations a continuously updated schedule of interventions.

In 2010 a new restoration of the main spire was started (Fig.1), presently extended on the flying arches and buttresses around the base of the main spire (Fig.2). The research shown here stems from such activities, requesting the analysis of the structural response of this part of the construction.

The structure under examination is made of stone masonry and includes iron members. The masonry is made of large blocks and other elements with varying shape, some elongated, some following the curved geometry of the arches. The metallic members are the arch ties (radial) and the circumferential ties in the core of the spire (Nava, 1845).

In addition, many connection “staples” (Beckmann and Bowles, 2004) are placed hidden within the masonry blocks: for example iron links to connect the elongated blocks along the arches (Fig.2) and vertical studs to connect the decoration tracery elements along the circumference. The staples are bent at their extremities and successively anchored with poured molten lead into the blocks cavities.

Figure 1. Main spire and scaffolding (VFD 2015).

The experimental observation highlights cracking that can be caused by load effects, by environmental actions such as corrosion, temperature variations and freeze-thaw cycles.

Rainwater is almost certain to find capillary paths to the iron and in combination with atmospheric oxygen cause rusting of the ironwork (Beckmann and Bowles, 2004). Corrosion cracks of a longitudinal metallic member form in planes originating from the member axis, and in the visual observations generally show on the outer surface along lines parallel to the corroding members.

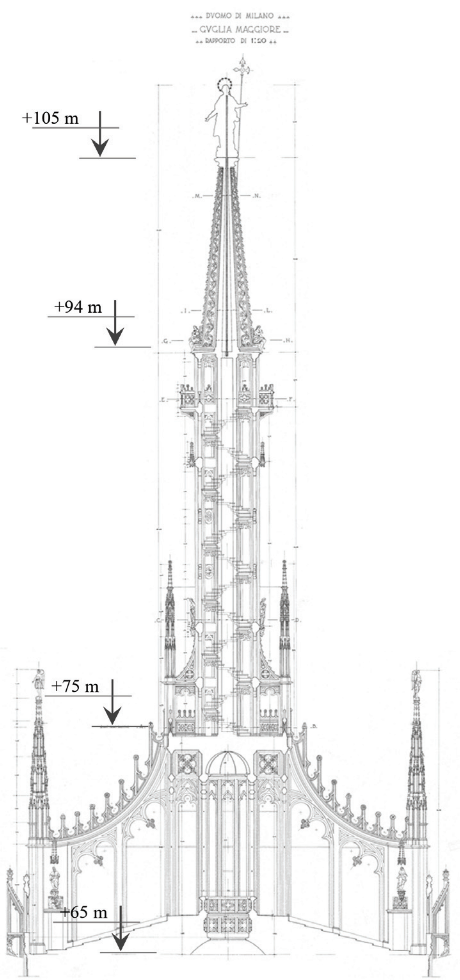

Figure 2. Section of the main spire of the Duomo along a radial plane (Ferrari da Passano 1988, Corradi & Calvi 2009).

For the structural analysis, the assumptions of Heyman's theory (Heyman 1995) for masonry consider no sliding, no tensile strength and unlimited compression strength, while in reality sliding may occur, the tensile strength is very small and the compression strength limited. In addition, no metallic members are considered in the formulation of this theory.

The aim of this paper is to analyse the structure at study combining experimental observations of the damage and limit analysis results of the structural action. The conclusions concern both the response of the construction and the methodology used.

2 DAMAGE EVOLUTION OF THE FLYING ARCHES

The main spire is a tall, acutely pointed octagonal pyramid, more than 40 meters high, placed over the cathedral masonry dome and ending on the top with the gilded "Madonnina" statue of the Virgin Mary, added in 1774 and reaching the total height of 108,5 m. (Figs 1–2). The octagonal base of the spire (at 75 m), just above the arches, is built of solid blocks of stone and radial iron ties. Beneath this, there are pillars arranged in two inner octagons, and eight radial reversed arches, connecting the spire to the exterior buttresses. The first octagon is composed by 4 couples of pillars, whether the second one of 8 pillars. The arches are supported between the spire and the buttresses on a third octagon of pillars.

The description of A. Nava of the main spire (Nava, 1845) referred to a huge metallic framework made of differently dimensioned elements, connecting small stone masonry pillars and arches. On the cathedral roof, the main spire is surrounded by eight delicate flying arches with pinnacles and statues.

The main spire of the Dome was built between 1765 and 1769 by Francesco Croce, after four centuries from the beginning of the construction, settling more than a century of projects and proposals, thus crowning the crossing of the lantern in the best possible way.

Just 70 years later, the serious decay of the main spire, mainly due to the corrosion of metallic parts, reached the peak in 1842 with the falling of some stone pieces from the "Belvedere", a balcony close to the top. After this event and the observation of its short durability, the architect of Veneranda Fabbrica, Pietro Pestagalli proposed demolishing and reconstructing the main spire under a new project developed by himself and more similar to the style of the Cathedral. The solution proposed by Ambrogio Nava in 1843, member of the Board of the Veneranda Fabbrica, was to evaluate more in detail all the damage. It was decided that restoration of the spire was possible without modifying its structure. Nava was sure of Croce's design quality, but found some construction defects that could be repairable.

In two years, starting from 1844, Nava repaired the damaged stone units, filling the cracks or substituting them, where necessary with new stones, making use of a hydraulic lime based mortar with pozzolana and marble powder. Where cavities were present, he inserted molten lead. He repaired the anchorages of the existing iron framework, where damaged, binding the ending parts with new iron elements inside the stones perforated on purpose. The internal volume of the spire was so seriously damaged that Nava had to substitute nearly all the metallic framework, element after element, adding a new system of vertical metallic elements (Nava 1845).

Despite the attention payed by Nava in the restoration of the main spire, also with the indication of preserving the metallic elements from the natural weathering, oxidation of the metal reinforcement structure started again, affecting the stability and causing breakage of many of the marble parts. In 1962, replacement of the metal frame with one made of stainless steel and the insertion of new marble elements in place of the damaged ones, made the Main Spire safe once again. Over the last 20 years, 25 spires have been restored, partially dismantled and re-built using a method similar to the like-for-like replacement of the wall facings (VFD 2015).

This intervention looked probably definitive for the metallic parts, but it could not avoid the degradation of marble parts, which somewhere appear heavily damaged. In 2010 the Veneranda Fabbrica del Duomo undertook again the task of restoring the marble of the main spire which was degraded by weathering, pollution and corroded tie rods. The planned restoration required the erection of scaffolding surrounding the spire almost to the top. This work became more delicate since it greatly increased the surface exposed to the wind. The structure was instrumented with a high variety of sensors to monitor wind conditions, position and movement of the scaffolding and spire by the Gicarus laboratory of Politecnico di Milano (Alba et al. 2011). A complete and time consuming 3D laser scanner survey was executed by Fassi et al (2011) of the Politecnico di Milano.

The huge spire is exposed to extreme weather conditions throughout the year. The entire restoration intervention, from assembly of the scaffolding to the preparation of the marble and the creation of replacement pieces, is being carried out by highly specialised personnel of the Veneranda Fabbrica who execute repair operations along a continuously updated schedule of interventions.

The main spire reversed arches show clearly the maintenance carried out along the years, with new stone elements placed to substitute the damaged ones and new mortars with different composition to repair the bedding joints and units detachments. The main cracks and detached parts are located on the stone units near the insertion of the tie rods, where the external and internal iron elements are seriously corroded (Fig. 3a,b).

Iron tie rods were seriously damaged since the end of the XVIII century due to weathering (rain water, thermal deformation, wind and pollution). Also different dynamics actions, like some seismic tremors, lightning may have affected over the years the efficiency of the tie rods. Weathering strongly affected the surface of the Candoglia marble units (Fig 4) and bedding mortar with deep erosion but other cracks typologies of structural nature are visible and in continuous evolution (Fig.5a,b).

In accordance with the main direction of the wind, North East—South West, the general crack pattern survey observed before the substitution of the iron tie rods, showed a difference among the reversed arches on the South side (AT1 – 4) and the ones on the North side (AT5 – 8):

a. the arches on the South side of the spire AT1 – 4 present longitudinal cracks on the long stone units forming the curved part (Fig. 6) as mainly visible in the arch named AT4. The cause of these cracks can be attributed to high compression stresses along the arches, combined sometime with the oxidation of the interior iron staples. In the North side arches, this crack typology is absent.
b. the vertical narrow pillars below the reversed arches, placed at mid span between the main spire and the external buttresses, present vertical cracks compatible with the high compression stress induced by the thrust line in the arches. (see the crack pattern survey in the section AT5–1 arches and section AT6–2 both North—South in figures 9 and 10). Arch AT6 in the North side, which is one of the most damaged ones, presents in addition horizontal cracks due to flexural stresses. This phenomenon demonstrates clearly that all these reversed arches,

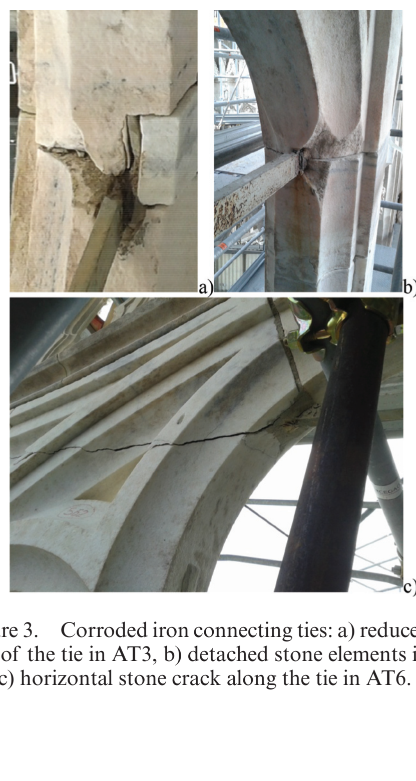

Figure 3. Corroded iron connecting ties: a) reduced section of the tie in AT3, b) detached stone elements in At1 and c) horizontal stone crack along the tie in AT6.

providing buttressing for the main spire, are constantly subjected to active actions.

c. the compressions thrust line of the reversed arches concentrates in the intrados near the just above described narrow pillars and the external buttresses elements. This phenomenon causes a tension stressed area in the extrados area, causing detachments from the stone units mainly concentrated on this area in all eight reversed arches. Repointed mortar joints are visible in the arches on the Southern side of the spire, while in the arches to the North, open mortar joints with a detachment measured around 1 cm is still clearly visible (Fig. 8ab).
d. all eight external buttresses of the reversed arches are cracked, mainly on the North side of the spire, on either the external or the internal side.

In Figs 9–10 the crack pattern survey is reported on some reversed arches to highlight the different condition in the opposite arches. Cracks are here emphasized in dimension to be readable at the reported scale.

Figure 4. Candoglia marble stone unit damaged by weathering in AT6.

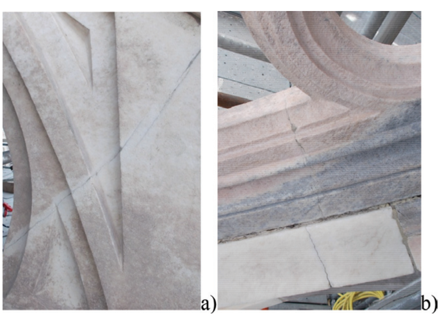

Figure 5a,b. Structural cracks on marble units: a) cracks on an old stone unit in AT6 and b) crack going through an old and a replaced marble unit in AT4.

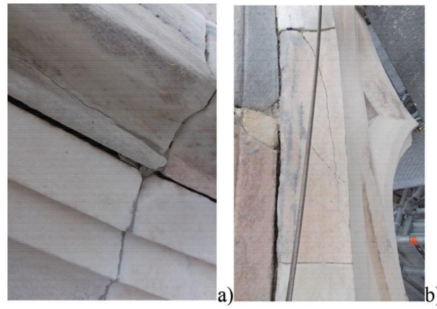

Figure 6. a. b. Longitudinal cracks due to compression stress: a) in AT3 and b) in AT4 arches.

Figure 7. Vertical cracks on small pillars in AT8.

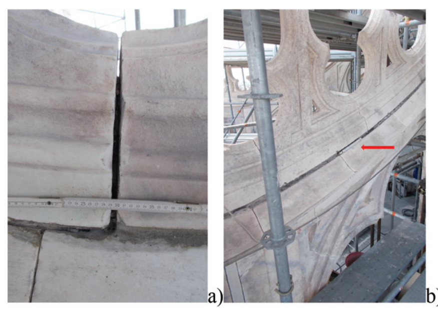

Figure 8. a. b. Detachment between stone units: a) at the extrados of the arches (AT7) and along the longitudinal units (AT6) where a wooden ruler is inserted in the bedding mortar.

3 LIMIT ANALYSIS

The object of study is the system of the spire base composed of the reversed arches, pillars, buttress and ties. Its gravity and wind loading are transferred to the 16 pillars beneath and the 8 reversed arches around the spire (Figs 1 and 13). The limit analyses carried out here are based on the lower-bound theorem, the so-called static approach, and the upper-bound theorem, the kinematic approach (Heyman 1995, Como 2013). The static method determines an equilibrium solution within the strength limits of the material. The thrust line obtained by the model can thus indicate the orientation of possible cracks parallel to the compression direction. A kinematic model indicates possible collapse mechanisms with the positions of "hinge" cross-sections with a part cracked in tension, and with compression in the other part; these correspond in the experimental reality to parts showing tensile and possibly also compression damage; the damage can be spread along a finite length rather than localized in one cross-section.

The kinematic model solution does not necessarily consider equilibrium, except for the real collapse mechanism that is also equilibrated. The closer the solution is to the real collapse mechanism, the more it will approach the equilibrium configuration.

Within the response of the structures to the loads, a number of hinges lower than those needed for collapse can initiate, showing some tensile and compression damage. Thus the examination of possible mechanisms, as close as possible to equilibrium, provides hints for the interpretation of the damage observed experimentally.

The spire is exposed to the wind, mainly in the NE-SW direction (according to the wind rose, Fig. 11).

The wind pressure is applied to the spire (level 75 m-108 m) according the national technical standards (NTC 2008) where the reference wind speed for Lombardy is 25 m/s and return period is 50 years.

The octagonal base of the spire, just above the arches, is built of solid blocks of stone with iron ties and thus considered as rigid. The spire base is connected to the substructure: its reactions resulting from the wind loading are transferred to the pillars and the reversed arches beneath (Fig. 12).

The substructure (Fig. 13) is composed of two inner octagons, laying directly beneath the spire, and eight radial reversed arches, connecting the spire to the exterior buttresses. For the construction of the thrust line the following assumptions are taken into consideration. The gravity load (normal component) of the spire is transferred to the 16 pillars beneath (first and second inner octagons).

The two remaining reaction components (shear and moment) at the spire base are the effects of the wind loading. The shear component is transferred to the reversed arches, opposing the wind loading "F" by the thrust "A" (Fig. 14). This scheme does not consider the contribution of the shear force of the pillars of the first and second octagon, and the tension in the tie on the side opposite to the movement of the base, anchored in the outer buttress.

The overturning moment produced by the wind must be equilibrated by forces at the support points under the solid base (the positions of the first and second octagon); these are determined by equilibrium calculations. Given the stiffness of the base it is assumed that the forces are proportional to the distance from each support point to the center of the

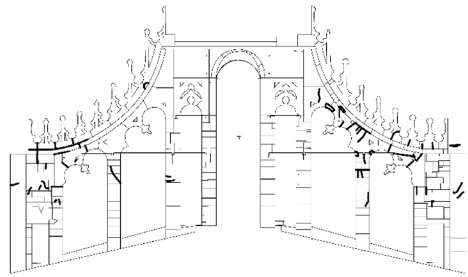

Figure 9. Survey of the arches AT5 (left) and AT1 (right).

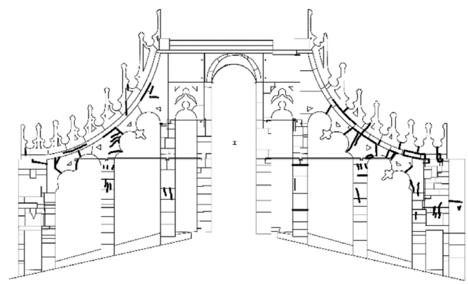

Figure 10. Survey of arches AT2 (left) and AT6 (right), direction South-North.

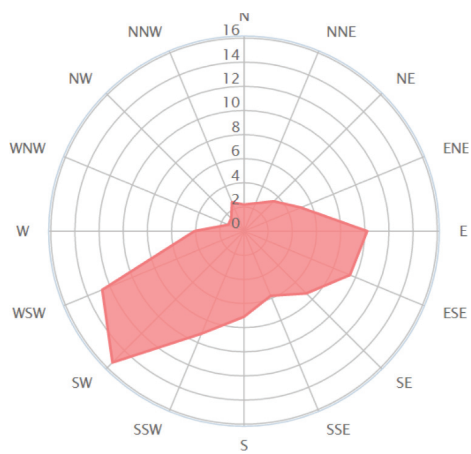

Figure 11. Annual Wind rose (%) [Milano-Linate meteorological station].

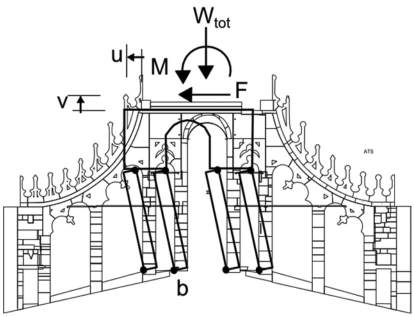

Figure 12. Cinematics of the spire base under spire equivalent loading.

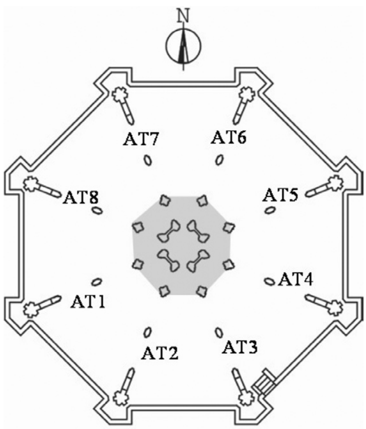

Figure 13. Main spire plan (inner octagon in grey).

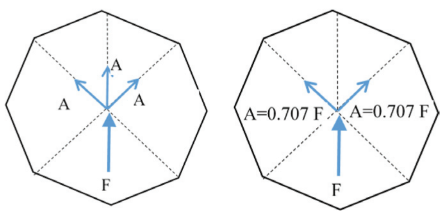

Figure 14. Wind load decomposition in the direction of arches for the 1st and 2nd configurations.

solid base (Fig. 14). It is assumed that the forces on the first octagon act on the pillars and those on the outer octagon act at the sides of the reversed arches.

Application of this simplified approach allows obtaining the equivalent loading action of the spire to the supporting base. The computed reaction components at the base of the spire (level 75 m) are Wtot = 2700 kN, F = 298 kN, M = 3781 kNm.

The most dangerous case is when the wind load (V = 298 kN) is acting in the arch direction. Within a lower bound limit analysis approach it is supposed that at the beginning the arch in the direction of the force is plasticized and after that the two other arches by its sides. It follows that 2.414 A = F, thus A = 123 kN.

During the repair operations described in the introduction, one arch at a time must be partly dismantled, to replace iron members with new metallic ties. Hence a second configuration is analysed, with one arch missing and the two arches on each side providing resistance to the wind loading. With a conservative approach, the same return period for the wind load was considered in both cases.

The thrust line analyses are carried out simplifying the geometry of the system as schematically drawn in Figure 15. The calculated forces are applied as V and H; a reaction force R is at the intermediate column support; the self-weight of the structure is applied as the P_i forces (Fig. 15). Historic drawings were examined and compared to a 3d laser scan results: a constant cross-section of the arch is determined, starting from the exterior buttress up to the connection with the main spire; here the cross section has a greater depth. Considering the strength of the stone masonry limited, an interior part of the cross-section within which the thrust line must lie is determined: greater eccentricity of the internal compression force would crush the masonry. One of the infinite equilibrated solutions is plotted in Figure 16. It is the one in accordance with the experimental damage observation in-situ.

The static limit analysis result shows that for the first configuration (where all the arches are active) an equilibrated solution is achieved, with the maximal compression 455 kN. In the second configuration the thrust line is within the limit of the reversed arches, with maximal compression 437 kN.

The loading horizontal and vertical components (H,V) transferred to the buttress for the first and second configuration are (185 kN,100 kN) and (345 kN,198 kN) respectively. The horizontal reactions of the arches are partly balanced by the iron ties. They are capable to support 115 kN in good condition and supposed to carry out just the half when heavily corroded (58 kN, determined on the basis of measured cross-sections and nominal cast iron strength).

The verifications of the buttresses were then carried out. For combined bending and axial force, the safety factor at the ULS (ultimate limit state) derived from the moment ratios (M_{Rd}/M_{Ed}) is 1.42. The second case is not verified as the ratio results 0.98 (even though it is close to 1 and within a safety margin of 5%). As a result, repair works had to be carried out with strengthening around the respective arches.

Also the kinematic solutions were obtained: two of the mechanisms worked out are shown in Figures 17 and 18. The position of the hinges was obtained by attempts, with the help of the variations in the depth of the cross section indicating probable positions. The ductility of the iron was assumed sufficient for plastic flow. The load mul-

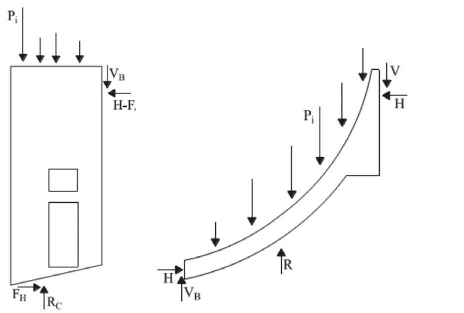

Figure 15. Equilibrated loads on buttress and reversed arches.

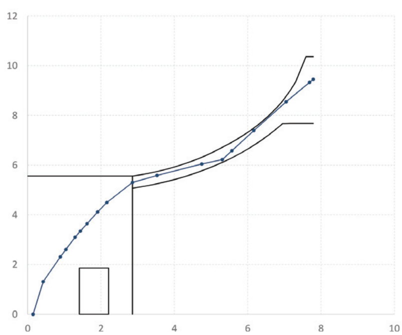

Figure 16. Thrust line within the reversed arch.

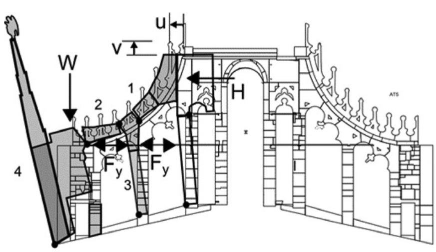

Figure 17. Theoretical mechanism 1 $\lambda = 2.96$ all arches are active (configuration 1); 1 $\lambda = 1.93$ one arch is deactivated (configuration 2).

tiplier in the case of one part of the iron tie yielding provided a reasonable upper bound to those obtained with the static approach and the buttress verifications. The system is now divided in several rigid blocks, connected together with hinges; the ties yield as the collapse mechanism is active. An imposed displacement and hinges in predefined points are necessary to activate a mechanism.

The first mechanism could take place with the yielding of the tie (Fig. 17). The cracks (hinges) that connect the blocks are formed in the parts with limited section. The load computed from this mechanism is nearly three times higher ($\lambda = 2.96$, 1st configuration) and two times higher ($\lambda = 1.93$, 2nd

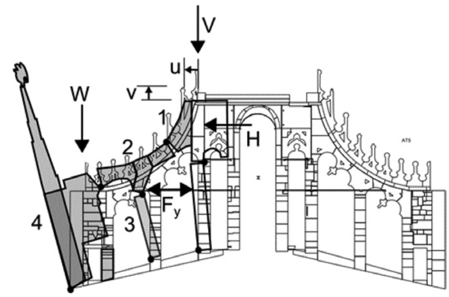

Figure 18. Theoretical mechanism 2; $\lambda = 1.65$ all arches are active (configuration 1); 1 $\lambda = 0.79$ one arch is deactivated (configuration 2).

configuration) than the computed wind load, determining thus an upper bound of the level of safety.

In the second collapse mechanism (Fig. 18) the load multipliers computed from this mechanism are $\lambda = 1.65$ (1st configuration) and $\lambda = 0.75$, (2nd configuration). Especially the second collapse mechanism lower than the unity shows the absence of the safety in the ULS (ultimate limit state). This result confirms once more the need for strengthening before proceeding with structural repairs.

4 DISCUSSION OF RESULTS

Stability of arch and buttress systems has been studied by Ochsendorf (2001), showing that either the buttresses fail, or the arches may collapse with large rotations developing a mechanism and the buttresses remain in position with limited displacements.

The limited damage of the buttresses examined in this study indicates their essential stability. There is more cracking in the arches, but this seems mainly related either to compressive strength being reached locally in some blocks or to corrosion phenomena. The position of the cracks is coherent with static thrust line analysis results. Some tensile cracking is visible as well as some sliding between blocks. Nevertheless hinges that could develop a mechanism in the arches do not appear evident in the Southern arches, while are more recognizable with their strong detachment in the tracery of northern arches.

The vertical cracks on the tracery above the arches may be mainly caused by corrosion of the vertical studs connecting these parts to the blocks of the arch, but some other cracks present an incompatible position and direction with the studs.

On the whole, mechanisms related to collapse conditions develop with extensive cracking; the examination of the cracking in this study does not indicate incipient collapse conditions. The observations on the whole regard a stable structure—reasonably far from its collapse.

5 CONCLUSIONS

The analysis of the flying reversed arches of the Cathedral of Milan main spire buttresses, was here presented. The limit analysis was based on the crack pattern survey observed before the recent restoration of the corroded iron connecting ties (in the second half of 2015). The aim of this analysis was to study the response of the flying arches to the stress induced by the wind and by the work of dismantling and replacement of the iron members with new metallic ties. This last operation required the discharge of a single arch, while its load was temporarily transferred to the adjacent arches. The predicted damage of the buttresses indicates their essential stability. The results confirm the structural role and the damage causes of the reversed arches. Weathering has a dominant role in the damage of the main spire (metal corrosion and stone detachment) but structural cracks are also evident due to the cyclic loads induced by the wind

The current restoration works are on-going and some of the main damages have been already repaired. The here reported crack pattern survey, no longer visible, and the analyses of the reversed arches describe a structural configuration that could be used as a reference to compare the eventual future structural damage of the reversed arches of the main spire. The prediction of the possible future damage could in addition avoid the achievement of a so serious stone blocks damage here observed, with the aim to reduce the necessary and regular operation of stone replacement and to reach a more sustainable future maintenance program.

The paper studies a masonry structure with iron ties and links connecting the blocks. This introduces some issues in the study of historic masonry structures, that are not frequently encountered. First of all, the analysis of the system has to take into consideration the presence of the ties. Secondly corrosion phenomena have to be taken into consideration. Finally, the iron links between blocks introduce a difference with respect to ordinary stone masonry with blocks and mortar, theoretically with a different composite action of the three components. For the limit analysis model the assumption made is that the iron has a sufficient ductility in order to develop a collapse mechanism where energy dissipation takes place in the ties and the tension force in the ties is considered in equilibrium solutions. It must be remarked that a development of the study should consider the possibility of the ties not being sufficiently ductile, hence studying equilibrium and collapse mechanisms without these members. This has sense also because the ties have localised corrosion that can limit their deformability. For the corroded ties, the effective cross-section with the localised reduction must be taken into account.

For the masonry behaviour, the effect of iron links theoretically limits the opening of cracks, leads to more energy dissipation and hence a less brittle behaviour and different localisation phenomena. These phenomena introduce new issue in the study of historic masonry structures.

ACKNOWLEDGEMENTS

This research was developed within the activities of the first year of "Convenzione Veneranda Fabbrica del Duomo e Politecnico di Milano". Special thanks to all staff at the Veneranda Fabbrica del Duomo especially Eng. F. Canali, Geom. F. Aquilano and Geom. M. Regis. Thanks to Prof. C. Achille and her research group, for the geometrical survey of the reversed arches and the students R. Ghiro and B. Mazzarotto for their help.

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